Spherical data analysis with R

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CONTENTS

CHAPTER

ONE

INTRODUCTION

This chapter introduces a number of functions to compute distance, direction, area and related quantities for spatial data with angular (longitude/latitude) coordinates. All of the functions are implemented in the R package `geosphere`. So to reproduce the examples shown you need to open R and install the latest verssion of the package `install.packages("geosphere")`.

It is generally easier to do such computations for data with planar coordinates. For example, with planar coordinates, ("Euclidian") distance can be computed with Pythagoras' theorem (a^2^+b^2^=c^2^), and the direction between a and b is ... Therefore, a common approach to compute certain quantities for angular data is to first transform ("project") the coordinates to a planar coordinate reference system (crs). However, such projections inevitably lead to distortions of one type or another (shape, distance, area, direction), and hence the quantities computed may not be accurate. The amount of disortion will vary, and may be minimal for smaller areas, as long an appropriate crs is used. For larger areas (for example with data covering a continent, or the entire world), it may be impossible, or impractical, to avoid marjor inaccuracies with this approach.

Computing quantities directly from the angular coordinates can be more precise, but it also has its shortcomings (that are shared with the planar approaches). Computations based on ellipsoids are refered to as geodesic computations. The main problem is that the earth has a irregular shape, which needs to be approximated to allow for the use of simple algoritms computations. The simplest approach is to treat the earth like a sphere – a three dimensional circle – defined by a radius. That allows for the use of spherical trigonometry functions. The shortest distance between two points on a sphere are on is part of a "great circle".

A more refined approach is to treat the earth like an ellipsoid (also termed sheroid), which is more accurate, as the earth is relatively flat at the poles, and bulges at the equator. Computations based on ellipsoids are refered to as geodesic computations; they are much more complex than spherical approximations. Ellipsoids are defined by three parameters: major axis a, minor axis b, and the flatting f (a small number, and therefore f is commonly expressed as the inverse flattening, 1/f. In geosphere, the default parameters for a, b, f are those that represent the ellipsoid used in the 1984 World Geodetic System (WGS84). This is a good average approximation for the entire world, but in more regional studies, more accurate computations may be possible using a locally preferred ellipsoid (as may be used by the national cartographic organization(s).

In this text, geographic locations are always expressed in longitude and latitude, in that order (!), because on most maps longitude varies (most) along the horizontal (x) axis and latitude along the vertical axis (y). The unit is always degrees, not radians, although many of the functions used internally transform degree data to radians before the main (trigonometric) computations take place.

Degrees are (obviously) expressed as decimal numbers such as (5.34, 52.12) and not with the obsolete sexagesimal system of degrees, minutes, and seconds. It is not hard to transform values between these two systems, but errors are commonly made. 12 degrees, 10 minutes, 30 seconds = 12 + 10/60 + 30/3600 = 12.175 degrees. The southern and western hemispheres have a negative sign.

dec2sex <- function(sd, latitude=TRUE) { d <- trunc(sd) r <- 60 * (sd - d) m <- trunc(r) r <- 60 * (m - r) s <- r / 3600 if (latitude) { if (d < 0) { h = 'S' } else { h = 'N' } } else { if (d < 0) { h = 'W' } else { h = 'E' } } paste0(d, 'd ', m, 'm ', s, 's ', h) }

```
sex2dec <- function(d, m, s, h=c(N,S,E,W)) { d + m / 60 + s / 3600 if (h %in% c('S', 'W') { d <- -d } return(d) }
```

CHAPTER

TWO

DISTANCE

2.1 Introduction

The geosphere package has six different functions to compute distance between two points with angular coordinates.

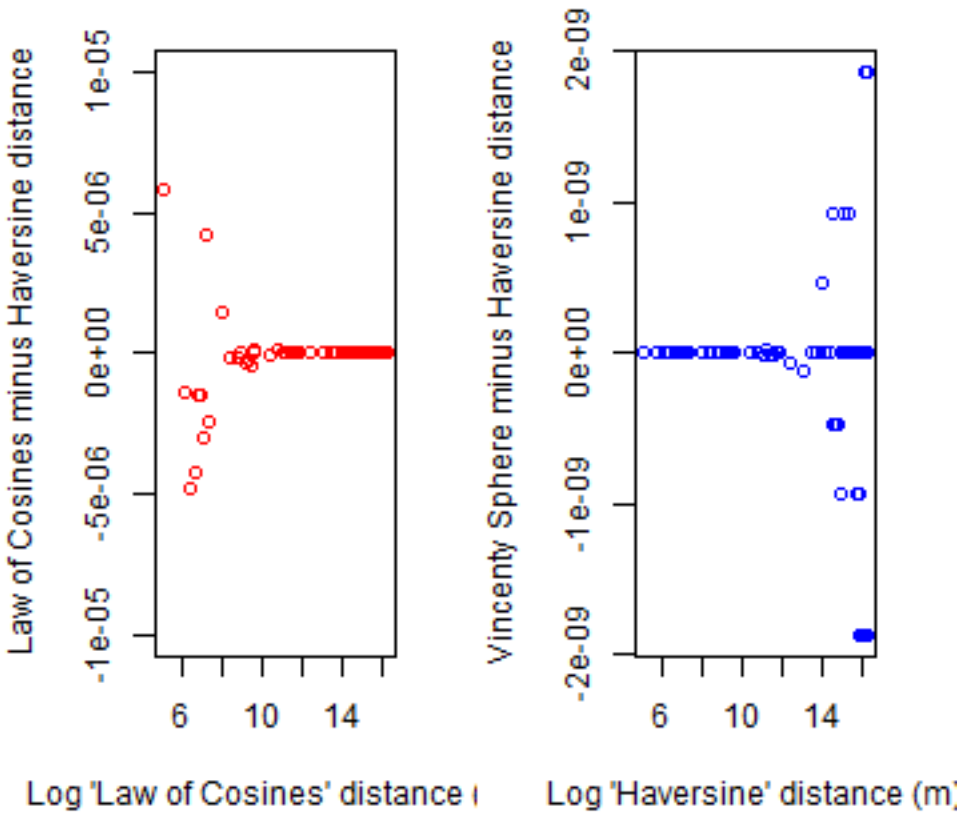
Three of these aproximate the earth as a sphere, these function implement, in order of increasing complexity of the algorithm, the 'Spherical law of cosines', the 'Haversine' method (Sinnott, 1984) and the 'Vincenty Sphere' method (Vincenty, 1975). The other three methods, 'Vincenty Ellipsoid' (Vincenty, 1975), Meeus, and Karney are based on an ellipsoid (which is closer to the truth). For practical applications, you should use the most precise method, to compute distances, which is Karney's method, as implemented in the `distGeo` function.

2.2 Spherical distance

The results from the first three functions are identical for practical purposes. The Haversine ('half-versed-sine') formula was published by R.W. Sinnott in 1984, although it has been known for much longer. At that time computational precision was lower than today (15 digits precision). With current precision, the spherical law of cosines formula appears to give equally good results down to very small distances. If you want greater accuracy, you could use the distVincentyEllipsoid method.

Below the differences between the three spherical methods are illustrated. At very short distances, there are small differences between the 'law of the Cosine' and the other two methods. There are even smaller differences between the 'Haversine' and 'Vincenty Sphere' methods at larger distances.

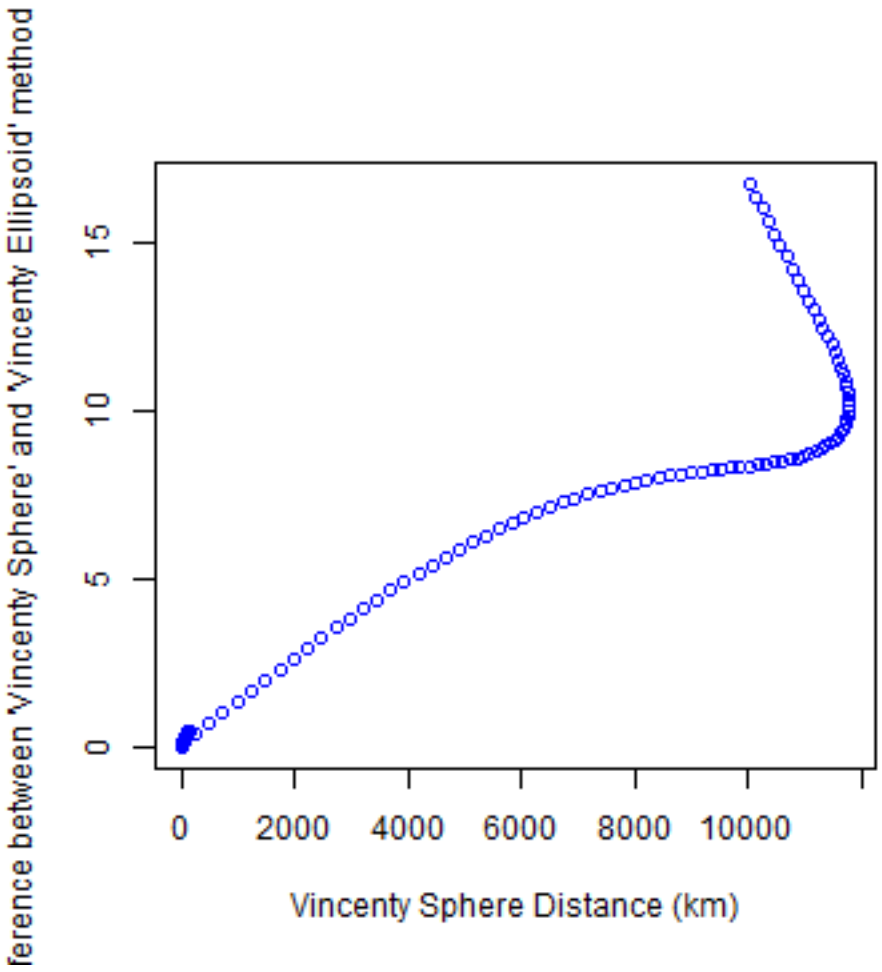
```
library(geosphere)
Lon = c(1:9/1000, 1:9/100, 1:9/10, 1:90*2)
Lat = c(1:9/1000, 1:9/100, 1:9/10, 1:90)
dcos = distCosine(c(0,0), cbind(Lon, Lat))
dhav = distHaversine(c(0,0), cbind(Lon, Lat))
dvsp = distVincentySphere(c(0,0), cbind(Lon, Lat))
par(mfrow=(c(1,2)))
plot(log(dcos), dcos-dhav, col='red', ylim=c(-1e-05, 1e-05),
            xlab="Log 'Law of Cosines' distance (m)",
            ylab="Law of Cosines minus Haversine distance")
plot(log(dhav), dhav-dvsp, col='blue',
            xlab="Log 'Haversine' distance (m)",
            ylab="Vincenty Sphere minus Haversine distance")
```



2.3 Geodetic distance

The difference with the 'Vincenty Ellipsoid' method is more pronounced. In the example below (using the default WGS83 ellipsoid), the difference is about 0.3% at very small distances, and 0.15% at larger distances.

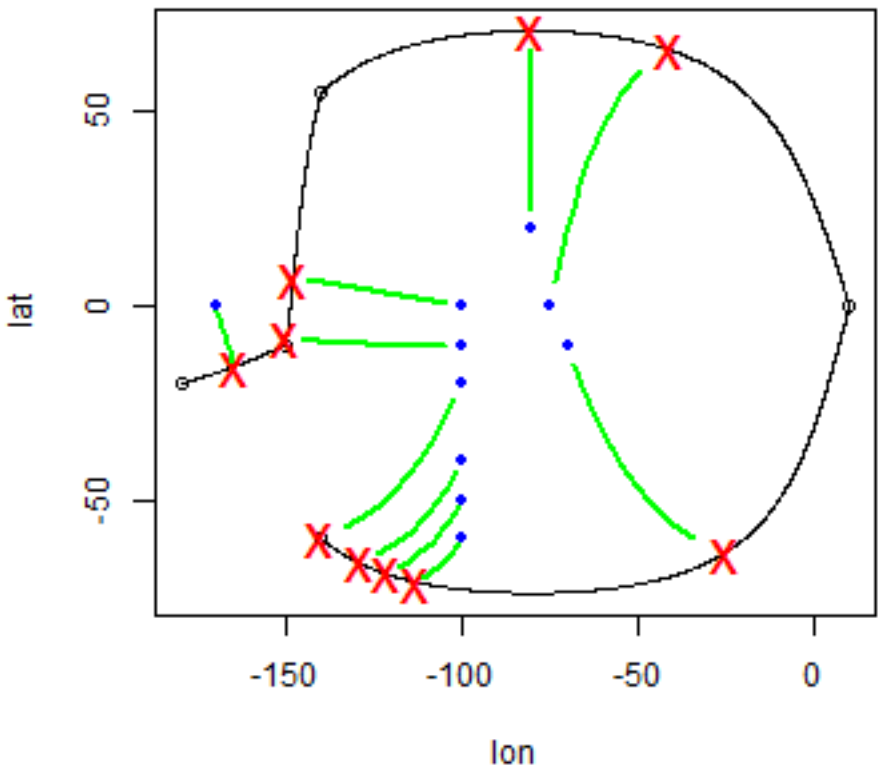
```
dvse = distVincentyEllipsoid(c(0,0), cbind(Lon, Lat))
plot(dvsp/1000, (dvsp-dvse)/1000, col='blue', xlab='Vincenty Sphere Distance (km)',
        ylab="Difference between 'Vincenty Sphere' and 'Vincenty Ellipsoid' methods
↪(km)")
```



2.4 Distance to a polyline

The two function describe above are used in the dist2Line function that computes the shortest distance between a set of points and a set of spherical poly-lines (or polygons).

```
line <- rbind(c(-180,-20), c(-150,-10), c(-140,55), c(10, 0), c(-140,-60))
pnts <- rbind(c(-170,0), c(-75,0), c(-70,-10), c(-80,20), c(-100,-50),
          c(-100,-60), c(-100,-40), c(-100,-20), c(-100,-10), c(-100,0))
d = dist2Line(pnts, line)
plot( makeLine(line), type='l')
points(line)
points(pnts, col='blue', pch=20)
points(d[,2], d[,3], col='red', pch='x', cex=2)
for (i in 1:nrow(d)) lines(gcIntermediate(pnts[i,], d[i,2:3], 10), lwd=2, col='green')
```



2.5 References

- Karney, C.F.F. GeographicLib.
- Sinnott, R.W, 1984. Virtues of the Haversine. Sky and Telescope 68(2): 159
- Vincenty, T. 1975. Direct and inverse solutions of geodesics on the ellipsoid with application of nested equations. Survey Review 23(176): 88-93. Available here.

CHAPTER

THREE

DIRECTION

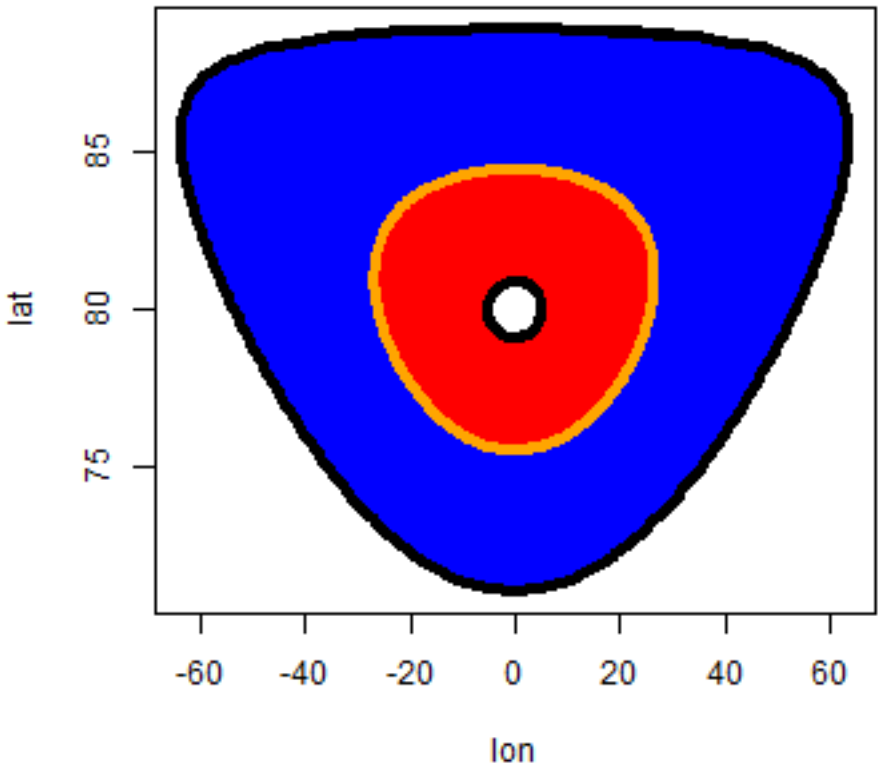
3.1 Introduction

3.2 Point at distance and bearing

Function destPoint returns the location of point given a point of origin, and a distance and bearing. Its perhaps obvious use in georeferencing locations of distant sitings. It can also be used to make circular polygons (with a fixed radius, but in longitude/latitude coordinates)

```
library(geosphere)
LA <- c(-118.40, 33.95)
NY <- c(-73.78,  40.63)

destPoint(LA, b=65, d=100000)
##            lon      lat
## [1,] -117.4152 34.32706
circle=destPoint(c(0,80), b=1:365, d=1000000)
circle2=destPoint(c(0,80), b=1:365, d=500000)
circle3=destPoint(c(0,80), b=1:365, d=100000)
plot(circle, type='l')
polygon(circle, col='blue', border='black', lwd=4)
polygon(circle2, col='red', lwd=4, border='orange')
polygon(circle3, col='white', lwd=4, border='black')
```



3.3 Triangulation

Below is triangulation example. We have three locations (NY, LA, MS) and three directions (281, 60, 195) towards a target. Because we are on a sphere, there are two (antipodal) results. We only show one here (by only using int[,1:2]). We compute the centroid from the polygon defined with the three points. To accurately draw a spherical polygon, we can use makePoly. This function inserts intermediate points along the paths between the vertices provided (default is one point every 10 km).

```
MS <- c(-93.26, 44.98)
gc1 <- greatCircleBearing(NY, 281)
gc2 <- greatCircleBearing(MS, 195)
gc3 <- greatCircleBearing(LA, 55)
data(wrld)
plot(wrld, type='l', xlim=c(-125, -70), ylim=c(20, 60))
lines(gc1, col='green')
lines(gc2, col='blue')
lines(gc3, col='red')

int <- gcIntersectBearing(rbind(NY, NY, MS),
            c(281, 281, 195), rbind(MS, LA, LA), c(195, 55, 55))
int
##              lon      lat      lon       lat
## [1,]  -94.40975 41.77229 85.59025 -41.77229
```

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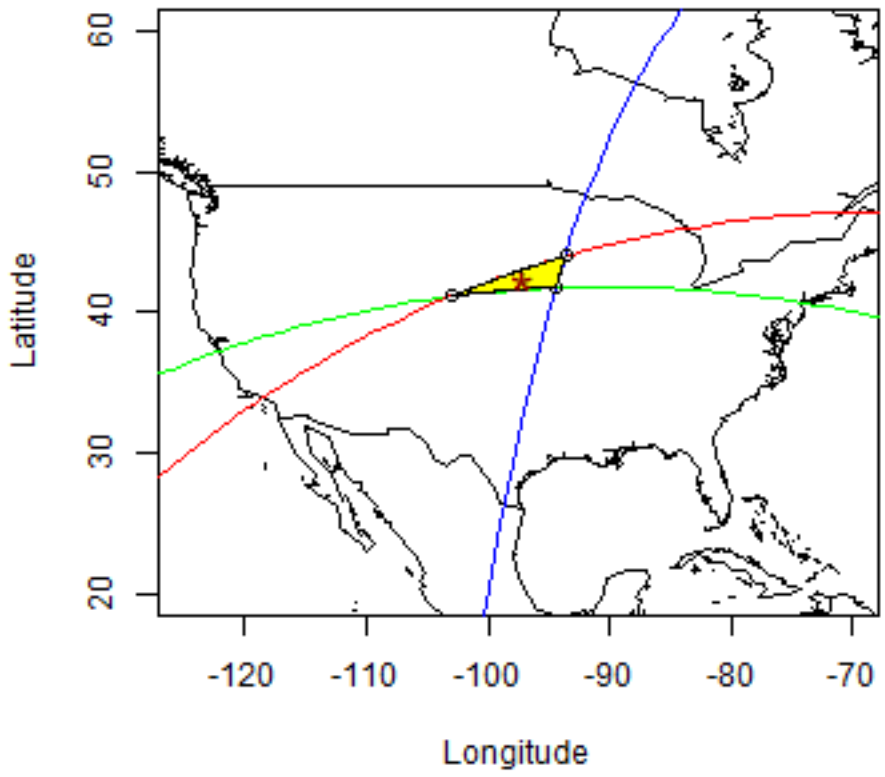
```
## [2,] -102.91692 41.15816 77.08308 -41.15816
## [3,]  -93.63298 43.97765 86.36702 -43.97765
distm(rbind(int[,1:2], int[,3:4]))
##             [,1]       [,2]       [,3]       [,4]       [,5]       [,6]
## [1,]        0.0   713665.0   253077.7 20003931.5 19308947.3 19751854.8
## [2,]   713665.0        0.0   823532.8 19308947.3 20003931.5 19195862.0
## [3,]   253077.7   823532.8        0.0 19751854.8 19195862.0 20003931.5
## [4,] 20003931.5 19308947.3 19751854.8        0.0   713665.0   253077.7
## [5,] 19308947.3 20003931.5 19195862.0   713665.0        0.0   823532.8
## [6,] 19751854.8 19195862.0 20003931.5   253077.7   823532.8        0.0
int <- int[,1:2]
points(int)
poly <- rbind(int, int[1,])
centr <- centroid(poly)
poly2 <- makePoly(int)
polygon(poly2, col='yellow')
points(centr, pch='*', col='dark red', cex=2)
```



3.4 Bearing

Below we first compute the distance and bearing from Los Angeles (LA) to New York (NY). These are then used to compute the point from LA at that distance in that (initial) bearing (direction). Bearing changes continuously when

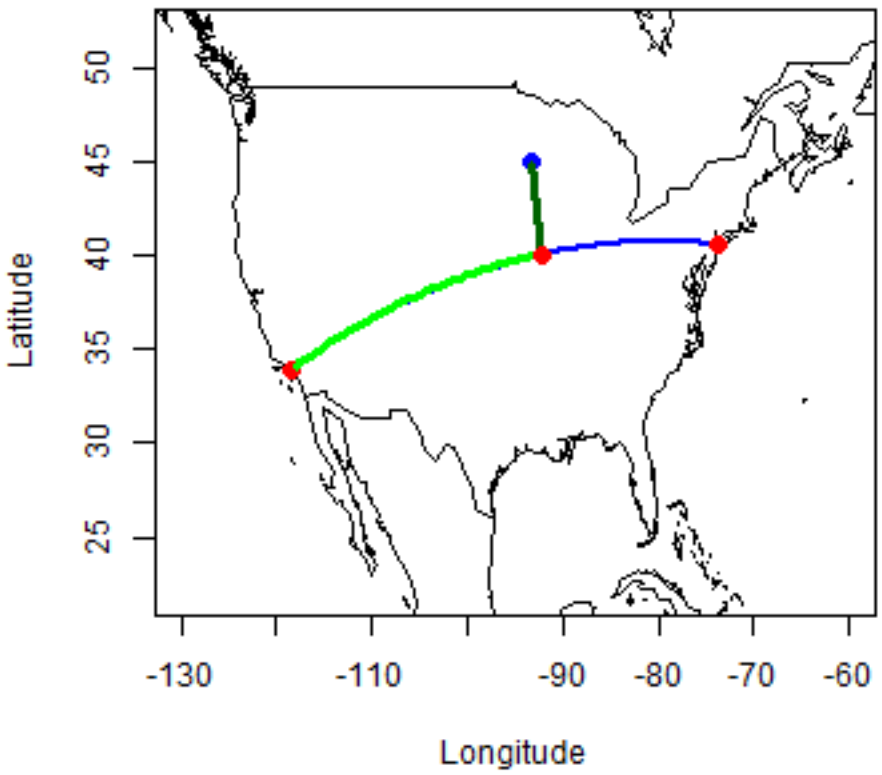
traveling along a Great Circle. The final bearing, when approaching NY, is also given.

```
d <- distCosine(LA, NY)
d
## [1] 3977614
b <- bearing(LA, NY)
b
## [1] 65.93893
destPoint(LA, b, d)
##             lon      lat
## [1,] -73.83063 40.63262
NY
## [1] -73.78  40.63
finalBearing(LA, NY)
## [1] 93.90995
```

3.5 Getting off-track

What if we went off-course and were flying over Minneapolis (MS)? The closest point on the planned route (p) can be computed with the alongTrackDistance and destPoint functions. The distance from 'p' to MS can be computed with the dist2gc (distance to great circle, or cross-track distance) function. The light green line represents the along-track distance, and the dark green line represents the cross-track distance.

```
atd <- alongTrackDistance(LA, NY, MS)
p <- destPoint(LA, b, atd)
plot(wrld, type='l', xlim=c(-130,-60), ylim=c(22,52))
gci <- gcIntermediate(LA, NY)
lines(gci, col='blue', lwd=2)
points(rbind(LA, NY), col='red', pch=20, cex=2)
points(MS[1], MS[2], pch=20, col='blue', cex=2)
lines(gcIntermediate(LA, p), col='green', lwd=3)
lines(gcIntermediate(MS, p), col='dark green', lwd=3)
points(p, pch=20, col='red', cex=2)
```




```
dist2gc(LA, NY, MS)
## [1] 549733.7
distCosine(p, MS)
## [1] 548738.9
```

3.6 Rhumb lines

Rhumb (from the Spanish word for course, 'rumbo') lines are straight lines on a Mercator projection map (and at most latitudes pretty straight on an equirectangular projection (=unprojected lon/lat) map). They were used in navigation because it is easier to follow a constant compass bearing than to continually adjust direction as is needed to follow a great circle, even though rhumb lines are normally longer than great-circle (orthodrome) routes. Most rhumb lines will gradually spiral towards one of the poles.

```
NP <- c(0, 85)
SF <- c(-122.44, 37.74)
bearing(SF, NP)
## [1] 5.15824
b <- bearingRhumb(SF, NP)
b
## [1] 41.45714
dc <- distCosine(SF, NP)
dr <- distRhumb(SF, NP)
```

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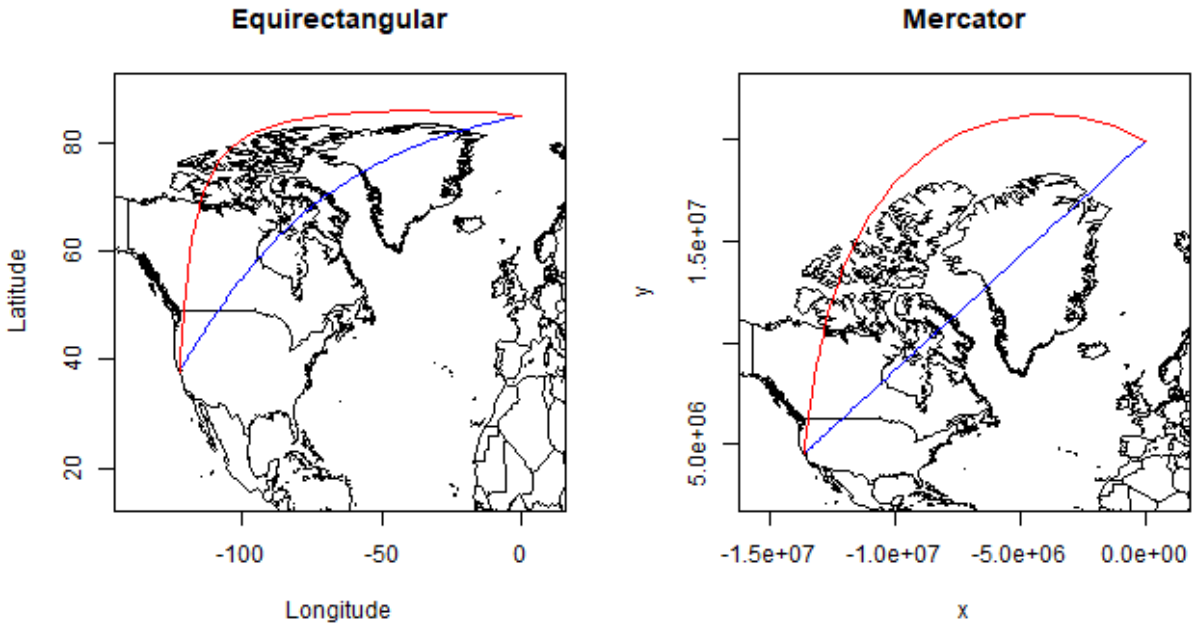
```
dc / dr
## [1] 0.8730767

pr <- destPointRhumb(SF, b, d=round(dr/100) * 1:100)
pc <- rbind(SF, gcIntermediate(SF, NP), NP)
par(mfrow=c(1,2))
plot(wrld, type='l', xlim=c(-140,10), ylim=c(15,90), main='Equirectangular')
lines(pr, col='blue')
lines(pc, col='red')
data(merc)
plot(merc, type='l', xlim=c(-15584729, 1113195),
                     ylim=c(2500000, 22500000), main='Mercator')
lines(mercator(pr), col='blue')
lines(mercator(pc), col='red')
```



CHAPTER

FOUR

TRACKS

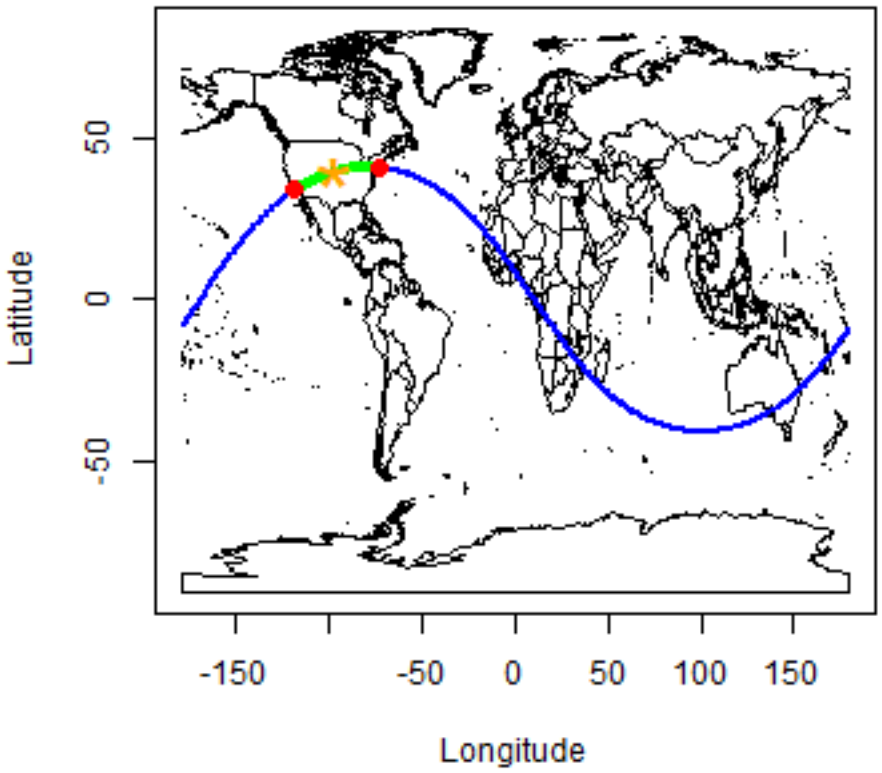
4.1 Introduction

Points on great circles, etc.

4.2 Points on great circles

Points on a great circle are returned by the function 'greatCircle', using two points on the great circle to define it, and an additional argument to indicate how many points should be returned. You can also use greatCircleBearing, and provide starting points and bearing as arguments. gcIntermediate only returns points on the great circle that are on the track of shortest distance between the two points defining the great circle; and midPoint computes the point half-way between the two points. You can use onGreatCircle to test whether a point is on a great circle between two other points.

```
LA <- c(-118.40, 33.95)
NY <- c(-73.78,  40.63)
data(wrld)
plot(wrld, type='l')
gc <- greatCircle(LA, NY)
lines(gc, lwd=2, col='blue')
gci <- gcIntermediate(LA, NY)
lines(gci, lwd=4, col='green')
points(rbind(LA, NY), col='red', pch=20, cex=2)
mp <- midPoint(LA, NY)
onGreatCircle(LA,NY, rbind(mp,c(0,0)))
## [1] FALSE FALSE
points(mp, pch='*', cex=3, col='orange')
```

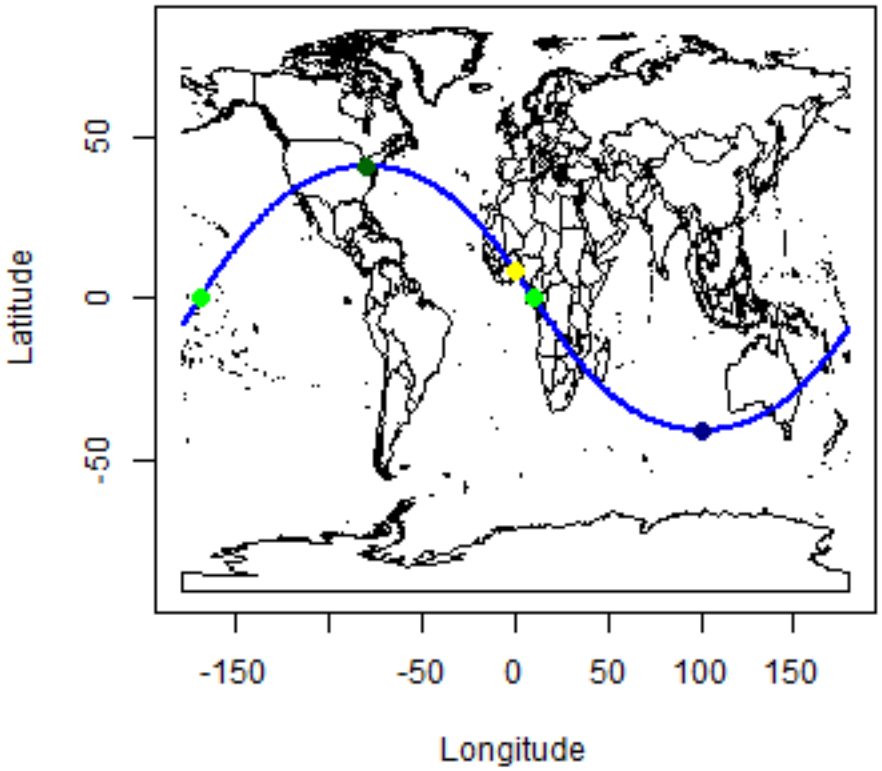



```
greatCircleBearing(LA, brng=270, n=10)
##         lon        lat
##  [1,] -144  31.210677
##  [2,] -108  33.532879
##  [3,]  -72  25.004950
##  [4,]  -36   5.254184
##  [5,]    0 -17.620746
##  [6,]   36 -31.210677
##  [7,]   72 -33.532879
##  [8,]  108 -25.004950
##  [9,]  144  -5.254184
## [10,]  180  17.620746
```

4.3 Maximum latitude on a great circle

You can use the functions illustrated below to find out what the maximum latitude is that a great circle will reach; at what latitude it crosses a specified longitude; or at what longitude it crosses a specified latitude. From the map below it appears that Clairaut's formula, used in gcMaxLat is not very accurate. Through optimization with function greatCircle, a more accurate value was found. The southern-most point is the antipode (a point at the opposite end of the world) of the northern-most point.

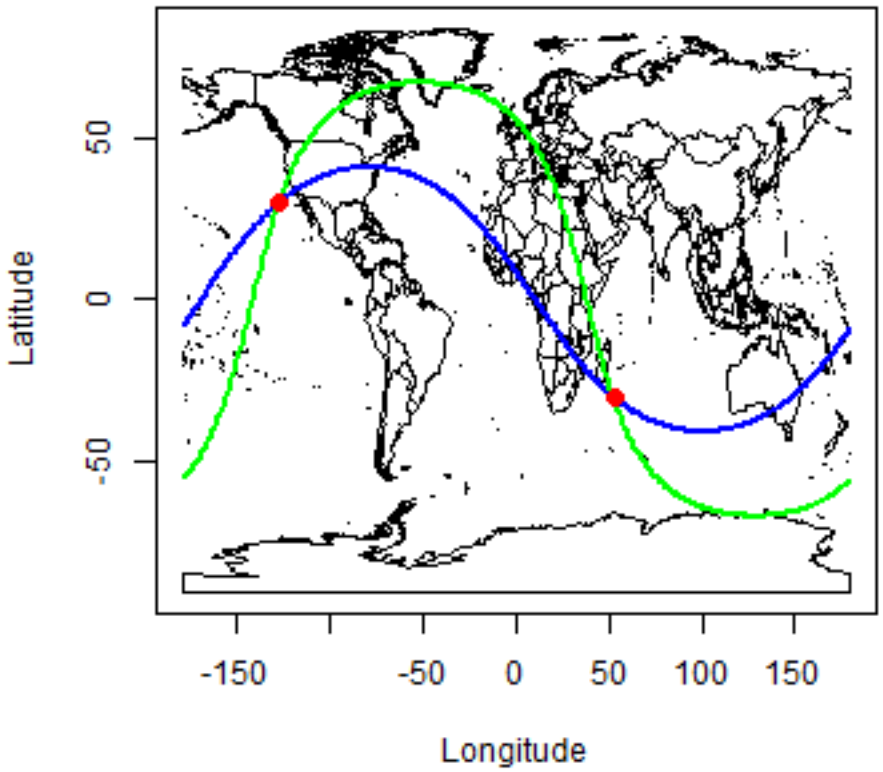
```
ml <- gcMaxLat(LA, NY)
lat0 <- gcLat(LA, NY, lon=0)
lon0 <- gcLon(LA, NY, lat=0)
plot(wrld, type='l')
lines(gc, lwd=2, col='blue')
points(ml, col='red', pch=20, cex=2)
points(cbind(0, lat0), pch=20, cex=2, col='yellow')
points(t(rbind(lon0, 0)), pch=20, cex=2, col='green' )

f <- function(lon){gcLat(LA, NY, lon)}
opt <- optimize(f, interval=c(-180, 180), maximum=TRUE)
points(opt$maximum, opt$objective, pch=20, cex=2, col='dark green' )
anti <- antipode(c(opt$maximum, opt$objective))
points(anti, pch=20, cex=2, col='dark blue' )
```



4.4 Great circle intersections

Points of intersection of two great circles can be computed in two ways. We use a second great circle that connects San Francisco with Amsterdam. We first compute where they cross by defining the great circles using two points on it (gcIntersect). After that, we compute the same points using a start point and initial bearing (gcIntersectBearing). The two points where the great circles cross are antipodes. Antipodes are connected with an infinite number of great circles.

```
SF <- c(-122.44, 37.74)
AM <- c(4.75, 52.31)
gc2 <- greatCircle(AM, SF)
plot(wrld, type='l')
lines(gc, lwd=2, col='blue')
lines(gc2, lwd=2, col='green')
int <- gcIntersect(LA, NY, SF, AM)
int
##          lon1      lat1      lon2     lat2
## [1,] 52.62562 -30.15099 -127.3744 30.15099
antipodal(int[,1:2], int[,3:4])
## [1] TRUE
points(rbind(int[,1:2], int[,3:4]), col='red', pch=20, cex=2)
```




```
bearing1 <- bearing(LA, NY)
bearing2 <- bearing(SF, AM)
bearing1
## [1] 65.93893
bearing2
## [1] 29.75753
gcIntersectBearing(LA, bearing1, SF, bearing2)
##          lon       lat       lon      lat
## [1,] 52.63076 -30.16041 -127.3692 30.16041
```

CHAPTER

FIVE

AREA OF POLYGONS AND SAMPLING

5.1 Area and perimeter of polygons

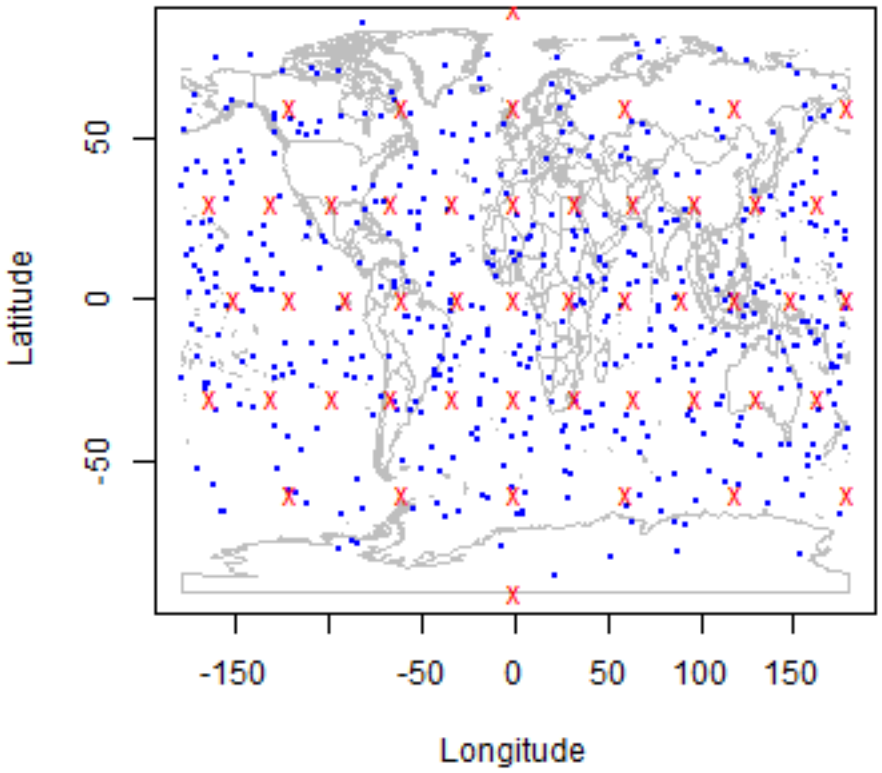
You can compute the area and perimeter of spherical polygons like this.

```
pol <- rbind(c(-120,-20), c(-80,5), c(0, -20), c(-40,-60), c(-120,-20))
areaPolygon(pol)
## [1] 4.903757e+13
perimeter(pol)
## [1] 27336557
```

5.2 Sampling longitude/latitude data

Random or regular sampling of longitude/latitude values on the globe needs to consider that the globe is spherical. That is, if you would take random points for latitude between -90 and 90 and for longitude between -180 and 180, the density of points would be higher near the poles than near the equator. In contrast, functions 'randomCoordinates' and 'randomCoordinates' return samples that are spatially balanced.

```
plot(wrld, type='l', col='grey')
a = randomCoordinates(500)
points(a, col='blue', pch=20, cex=0.5)
b = regularCoordinates(3)
points(b, col='red', pch='x')
```


Latitude
50
0
-50
-150
-50
0
50
100
150
Longitude